

A method to determine the charge of particles stopping in nuclear emulsions

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With 12 figures in the text

SUMMARY

An investigation is described of a photometric method for the determination of the charge of particles stopping in nuclear emulsions. The image of the track in the emulsion was projected on a narrow slit placed in front of a photomultiplier and moved perpendicularly across the slit. The slit was parallel to the direction of the track and the absorption of light was measured by means of the photomultiplier and a recording instrument. The resolving power depends on the available track length, the quality and the degree of development of the emulsions. If track lengths of $100\ \mu$ are used in normally developed G 5 emulsions of good quality it is possible to separate doubly and singly charged particles in about 95 % of the cases, the resolution being better for multiply charged particles.

Introduction

The correct interpretation of many events in nuclear emulsions depends on whether it is possible to determine the charge and the mass of particles with low energy and short range. As a rule emulsions of low sensitivity give the best resolution between different charges. But in many experiments normally developed electron sensitive emulsions must be used with appreciable difficulties in charge identification as a consequence. The reason for these difficulties is the small amount of useful information which can be obtained from a dense track. The number of gaps is very small; the δ -rays are short and very few and the grain density is high, making grain counting impossible. In spite of the difficulties several ways of charge determination have been tried.

The number of δ -rays has been counted in specially prepared emulsions (1) but since no δ -rays can be observed when the residual range is less than about $50\ \mu$ the method is not very useful for short tracks. The thin down length can be used, in theory, for the identification of the charge (2) but the method seems to be too unreliable. Measurements of the track area or a photometrically defined "half width" of the very last part of a track apparently give valuable information in many cases (3, 4, 5, 6, 7, 8).

However, several of these and most other methods are more or less influenced by subjective estimates and therefore the measurements must be performed with

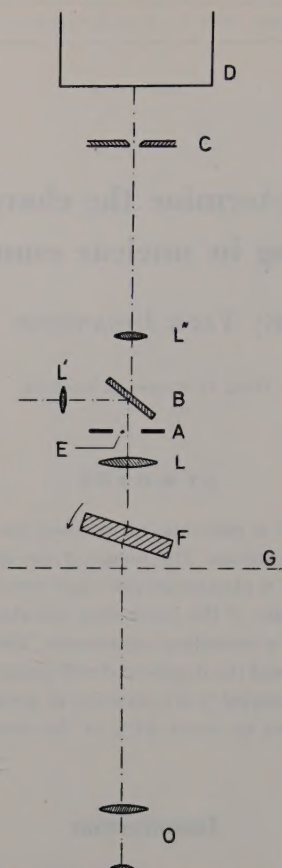


Fig. 1. Principle of the photometer.

excessive care; and in order to be reproducible it is often necessary for them to be repeated by the same person.

To avoid such difficulties an attempt has now been made to measure photometrically the projected areas of the tracks near their ends to see whether it is possible to obtain a parameter, sensitive primarily to the charge and if possible to the mass also.

The photometer

The main features of the photometer are shown in Fig. 1. It is a modification of the photometer described by von Friesen and Kristiansson (9), and Kristiansson (10). The track in the emulsion is projected in a plane at (A) by means of the objective (O) and the lens (L). The image at (A) can be regarded through the eye lens (L') by means of a plane-parallel glass plate (B) whose glass surfaces act as a mirror permitting most of the light to pass through it. The image at (A) is again projected onto a slit (C) by the lens (L'). Above the slit there is the cathode (D) of a photomultiplier which is of the end-window type (Du Mont 6292). The diameter of the cathode is much larger than the length of the slit. This type of photomultiplier permits a large total magni-

fication of the image and therefore the dimensions of the slit need not be inconveniently small. The dimensions of the slit are $6.6 \text{ mm} \times 0.5 \text{ mm}$ corresponding to $10 \mu \times 0.76 \mu$ in the object plane. The dimensions are not so important but especially the width must be kept constant for a series of measurements if the individual measurements are to be directly compared. The reasons for the choice of the slit dimensions described here will be given in connection with the discussion of the correction for dipping tracks. At the first image there is a thin wire (E) parallel to the slit and serving to facilitate the adjustment of the track parallel to the slit. The image seen through the eye lens is double because of the twofold reflection of the glass plate (B); but this is not very inconvenient since one of the images is less sharp than the other and the spacing between them is large, the glass plate being about 2 mm thick. The lateral displacement of the image across the slit is brought about by another 4.5 mm thick plane-parallel glass plate (F), the rotation of which is performed by a small synchronous motor mounted on the photometer. The angular speed is chosen in such a way that a suitable speed of displacement of the image across the slit is obtained. This speed is chiefly limited by the time constant of the registering device. The upper part of the photometer is revolvable around a bearing at (G) and since it would entail a rather heavy load on the microscope the whole photometer is counterbalanced by a weight.

The output from the photomultiplier is fed through a DC-amplifier to a Speedomax penrecorder. The paper speed is about 1.5 mm/sec., and the speed of displacement of the track is equivalent to about $0.1 \mu/\text{sec.}$ in the plane of the emulsion.

When the glass plate (F) is in its initial position its normal deviates about 10° from the optical axis of the microscope. If it is turned 20° this is equivalent to a displacement of the track in the emulsion of nearly 6μ , which is more than enough to give the absorption curve of a track. Since the rotation takes place symmetrically around the zero-position of the glass plate, the non-linearity of the displacement is very small. Because the reflection of light is very nearly constant the illumination of the slit is always the same, provided the illumination of the object plane is uniform.

Before the measurements started, the photometric instrument was tested by methods earlier described by Kristiansson (11).

A measurement is performed in the following manner. The track is adjusted parallel to the slit by means of the thin wire in the object plane of the eye-piece so that the track lies outside the slit. The synchronous motor and the motor driving the paper of the penrecorder are now switched on. The image of the track moves across the slit and an absorption curve is drawn on the paper. See Fig. 2. The straight line AB is drawn in such a way that it satisfactorily fits the absorption curve immediately outside the track. The area ABC is a function of the number of grains in the track and therefore a measure of the ionization caused by the particle. This area is divided by the length u , thereby making the former independent of the intensity of the light and the gain of the photomultiplier. The area can be accurately found by means of a planimeter, but this is a tedious procedure when many measurements have to be analysed. Instead, a simpler method has been used without loss of accuracy. The quantities u , h (the height of the peak) and b (the half width) were measured. The expression $h \cdot b/u = \text{MTW}$ (the mean track width) was found to be proportional to the area ABC divided by the length u . MTW has the dimension of length. When all measurements are performed under the same conditions it is only the relative "lengths" which are of importance. The MTW therefore, has been treated here without reference to its dimension.

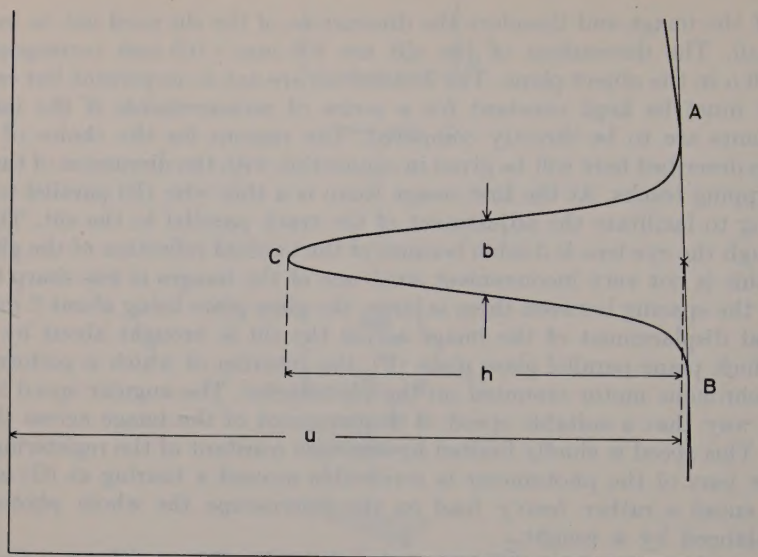


Fig. 2. Absorption curve for a proton track near the end of its range. From the figure it can be seen that there is a slight gradient in the background absorption. Since the line AB is fitted to the curve as close to the peak as possible the value u is a good measure of the background transmittance behind the track.

As long as the slit is narrower than the track the general shape of the absorption curves for tracks near the ends of their ranges is very nearly the same. This is because the number of δ -rays is negligible. And, therefore, the ratio of the area, as measured with a planimeter, to the product $b \cdot h$ should be nearly constant. This has been confirmed by experiments.

Selection of tracks, correction for depth and dip in the emulsion

The tracks on which the investigation was performed were found in G 5 emulsions exposed by the Sardinian expedition in 1953, and in G 5 plates exposed over Texas, USA, in 1955. The thickness of the emulsions was 600μ and the plateau values were 20 and 19 blobs/100 μ , respectively.

It seemed advisable to test the present method on emulsions of different quality. The Sardinian plates were of very good quality but those from Texas had a large number of slow electrons as a background.

Most of the selected tracks had residual ranges long enough to make possible a good estimate of the charge already by visual means only. If the assumed charge was 1 or 2 tracks with a residual range $\geq 200 \mu$ were used, if it was ≥ 3 tracks with $R \geq 50 \mu$ were chosen.

All measurements of MTW must be corrected for depth in the emulsion. About ten tracks of singly charged particles, ending in the emulsion, situated at various depths and with dip angle about zero are chosen. On these tracks the last 200 μ are measured. The MTW can for this special purpose be considered as constant over the

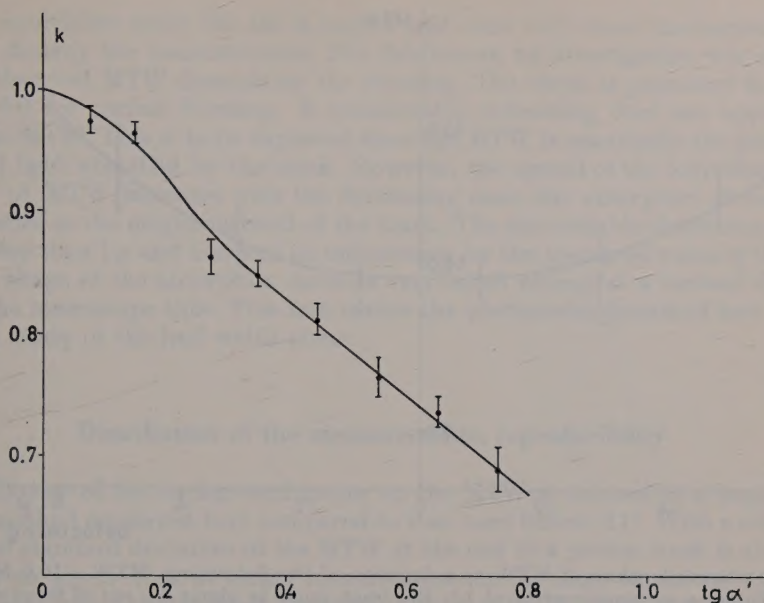


Fig. 3. The dip correction factor, k , as a function of the dip angle, α' , in the processed emulsion.

whole of this range. When the MTW-values are plotted versus depth in the emulsion the correction factor is immediately found (6, 11). The uncertainty of this factor will amount to about 1–1.5 %, which is of little importance for the charge determination. The correction factor found here is the same as that which must be applied to the measurements by the photometer mentioned earlier (9, 10).

If the tracks are inclined, relative to the plane of the emulsion, the observed values of MTW will be larger and a correction for dip must be applied. The explanation of the increase in MTW with the dip angle is the following:

1. The projected length of a track is shorter than the real length.
2. The shrinking of the emulsion condenses the track.

The outcome of each of these two effects is that the number of grains per projected unit length will be larger. In theory it is possible to obtain a formula for the correction showing the dependence of MTW on dip angle, grain density and shrinkage factor. In practice it is very difficult to find this formula. In this investigation the correction for dip was determined empirically, this being the easiest way since the shrinkage factor need not be accurately known and no special assumptions about the behaviour of the grains during the shrinkage process are required. Tracks with different dip angles have been measured and from each track a point on the correction curve can be obtained for the corresponding dip angle. In Fig. 3 the correction, k , is represented as a function of $\text{tg } \alpha'$, where α' is the dip angle in the processed emulsion. The correct MTW-values are calculated from the formula $\text{MTW}_{\alpha'=0} = k \cdot \text{MTW}_{\alpha'}$. As soon as the number of grains is so large that there are only a few gaps in the track this correction factor seems to be almost independent of MTW so that it could be applied to the whole range of MTW considered here. It seems feasible to measure tracks which are inclined to about 40° in the processed emulsion corresponding to about 65° in the original

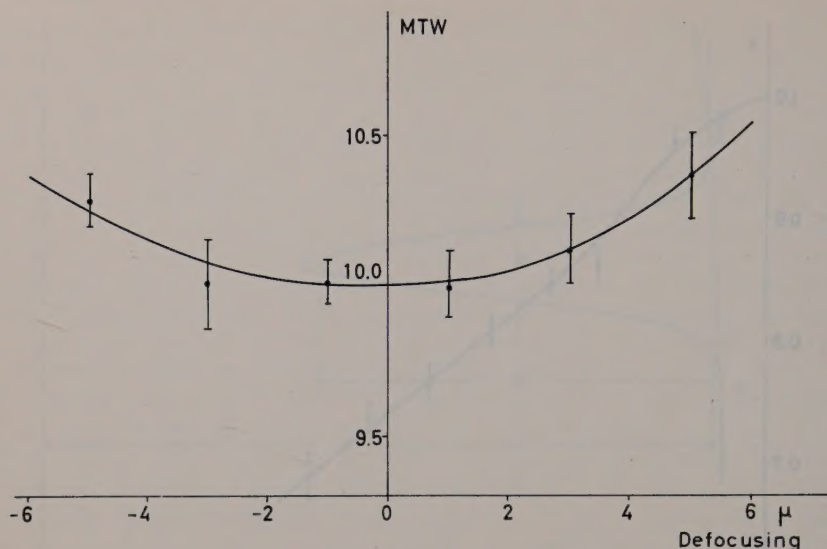


Fig. 4. The observed values of MTW as a function of the defocusing. MTW = 10.0 for correct focusing. To get a systematic error of 1 % the track must lie about 3μ out of focus which will never occur in practice.

emulsion since the shrinkage factor was about 2.3. Of course, all track lengths must refer to the unprocessed emulsion. The projected lengths have all to be multiplied by the factor $(1 + x^2 \text{tg}^2 \alpha')^{\frac{1}{2}}$, where x is the shrinkage factor of the emulsion.

Slit dimensions

The dimensions of the slit were $10 \mu \times 0.76 \mu$ in the plane of the emulsion. The reasons for choosing the above dimensions for the slit are:

1. A long slit means a lower number of measurements but is unsuitable for tracks having large dip angles because the correction for dip is a function of the slit length also. Furthermore, the use of different slit lengths would mean very much work in determining the dip correction factor, which must be accurately known. The use of the short slit described here permitted throughout the whole investigation the study of very steep tracks and the use of the correction factor given in Fig. 3.

2. A narrow slit will give more detailed information about a track than a broad one. But the amount of light is limited and a very narrow slit would mean too much shot noise in the output current from the photomultiplier. The width 0.76μ turned out to be a good compromise, the slit being narrower than the tracks studied but yet wide enough to give sufficient amount of light on the photocathode.

Influence of the focusing

Every single measurement requires considerable time, about one minute. During this relatively long period it may be difficult to keep the track perfectly focused on the slit. No adjustments are possible during the time when the track is "seen" by

the photomultiplier since the slit is narrow and even very small mechanical vibrations will destroy the measurements. For this reason an investigation was made of how the observed MTW depends on the focusing. The result is presented in Fig. 4. $MTW = 10.0$ for correct focusing. A considerable defocusing does not appreciably change the MTW. This is to be expected since the MTW is essentially the integrated amount of light absorbed by the track. However, the spread of the individual measurements of MTW increases with the defocusing since the absorption of the background varies in the neighbourhood of the track. The unavoidable defocusing is very seldom more than 1μ and hence of no importance for the measured value of the track area. The shape of the absorption curve is very much altered at a vertical displacement of the microscope tube. This fact makes the photometer described here unsuitable for a study of the half width alone.

Distribution of the measurements, reproducibility

The influence of the background grains on the MTW is reduced to a large extent with the method employed here compared to that used before (11). With a slit length of 10μ the standard deviation of the MTW at the end of a proton track is about 9 % compared to about 15 % at earlier measurements in the same emulsions. Since the spread arising from the statistical variation of the number of grains cannot be reduced it seems plausible to assume that the error from the background is small. The reason for this reduction of the influence of the background is that the track is compared to the background immediately outside the track in the manner shown in Fig. 2.

The reproducibility in the measurements is good. If the last 200μ of a proton track are measured repeatedly (the slit length being 10μ , 20 measurements are required), the standard deviation of the integrated MTW will amount to about 1 %.

Method of identification

Figs. 5 A–C show $\sum_0^R MTW$ where the summation has been extended from $R = 0$ to $R = 200\mu$, 100μ , and 50μ , respectively. The slit length was always 10μ . All measurements have been normalised so that the mean value of $\sum_0^{100\mu} MTW$ has been put equal to 100 for singly charged particles.

If the residual range 200μ is used (Fig. 5A) the measurements divide nicely into three groups. There can hardly be any doubt that in this case the separation between different charges is complete.

In Fig. 5B we see the total track area of the last 100μ of the same tracks. The number of particles with $Z = 1$ is 59, 29 have $Z = 2$ and 4 have $Z = 3$. In this case too a clear splitting into three different groups is obtained although the separation is no longer a perfect one. However, should the total track area fall just midway between two main groups, the separation may in some cases be improved, if the following fact is also considered. A track whose charge is wanted should be compared with tracks in the same area of the emulsion. In the present investigation this did not give more information in the doubtful cases, the reason for this being that the plates did not show any marked irregularities.

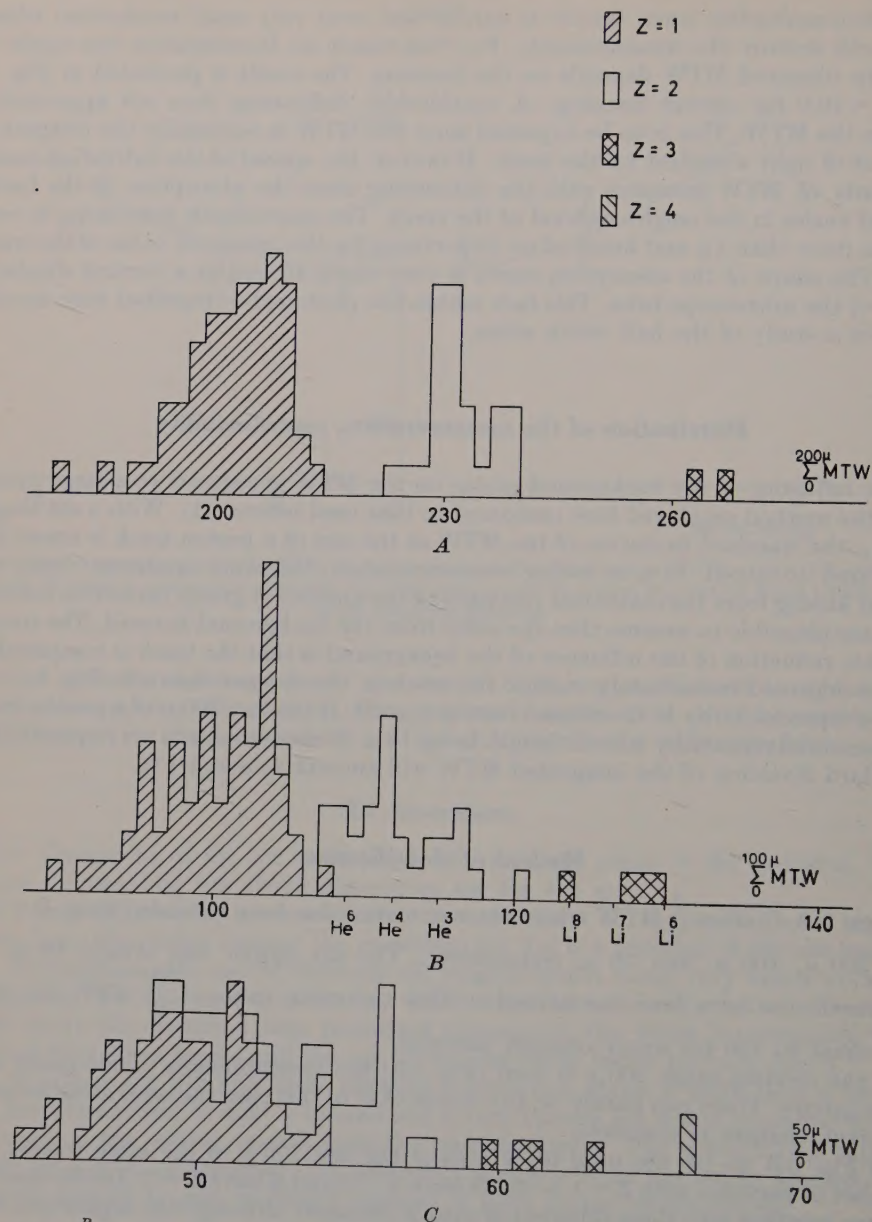


Fig. 5. A-C. $\sum_0^R MTW$ for different residual ranges. A. Histogram for $R = 200 \mu$ showing a complete separation between different charges. B. The corresponding histogram when 100μ has been used. One singly charged particle out of 59 belongs to the group with $Z = 2$ and one doubly charged out of 29 particles belongs to the group with $Z = 1$. The places where the measurements on some isotopes are expected to fall are indicated in the diagram. Because of the statistical spread of the measurements of He^4 it is impossible to tell whether any He^3 or He^6 nuclei are present. The measurement at $MTW = 124$ is of a Li^8 . C. If only a 50μ length has been used it is very difficult to separate charges 1 and 2. Higher charges seem to be more certain.

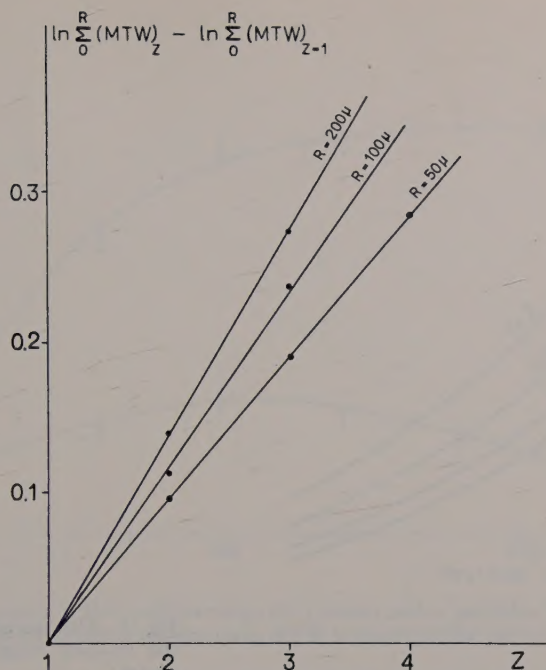


Fig. 6. $\ln \left(\sum_0^R \text{MTW} \right)_Z - \ln \left(\sum_0^R \text{MTW} \right)_{Z=1}$, plotted as a function of Z and with the residual range R as a parameter. In this representation the measurements very nearly fall on a straight line.

In diagram 5B one track area is found at the value of 124. It has been identified as a Li^8 by its decay. Measurements on tracks very close to this event did not show any anomalous results. However, the track made by the lithium ion has the correct area if we also consider the effect on ΣMTW caused by different isotope masses which will be discussed later.

If only the last 50μ are used, it seems in most cases impossible to separate charge 1 and 2 with a reasonable degree of certainty. The ratio of the number of particles with $Z = 1$ to the number with $Z = 2$ can be estimated fairly accurately. The higher charges are more certain.

Estimate of the uncertainty in Z

If $\ln \left(\sum_0^R \text{MTW} \right)_Z - \ln \left(\sum_0^R \text{MTW} \right)_{Z=1}$ is plotted as a function of Z a straight line is obtained. Fig. 6 shows this relation with the residual range as a parameter. The equation for such a line can be written

$$\ln \left(\sum_0^R \text{MTW} \right)_Z - \ln \left(\sum_0^R \text{MTW} \right)_{Z=1} = K(R) \cdot (Z - 1) \quad (1)$$

where $K(R)$ is the slope of the straight line. If $n(R)$ is the number of measurements (in the present case with a 10μ slit), we obtain for the error in Z

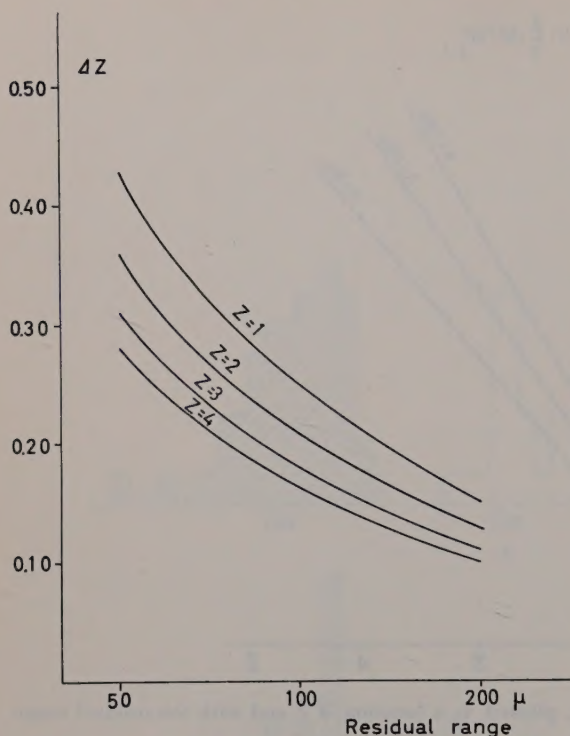


Fig. 7. The uncertainty in Z versus the residual range with Z as a parameter.

$$\Delta Z = \frac{\sigma(\text{MTW})}{K(R) \sqrt{n(R)}} \quad (2)$$

$\sigma(\text{MTW})$ = relative standard deviation of MTW.

In the above formula the error arising from the determination of $\left(\sum_{z=1}^R \text{MTW}\right)_{z=1}$ and $K(R)$, has been neglected. Since these two quantities have been determined from a large number of measurements the uncertainty in them is indeed very small, contributing less than 5 % to the total error in Z . $\sigma(\text{MTW})$ varies slightly with MTW. The dependence on MTW is given approximately by Table I. It has been calculated from measurements of different MTW.

In Fig. 7 the estimated uncertainty ΔZ is given as a function of the residual range and with Z as a parameter.

MTW-range relations for different particles

In Fig. 8 the MTW is shown as a function of the residual range for protons and α -particles. The experimental points of the two curves are the mean values from 18 protons identified by gap counting and the same number of α -particles, taken from the group in Fig. 5A and assumed to consist of particles with charge 2. For protons

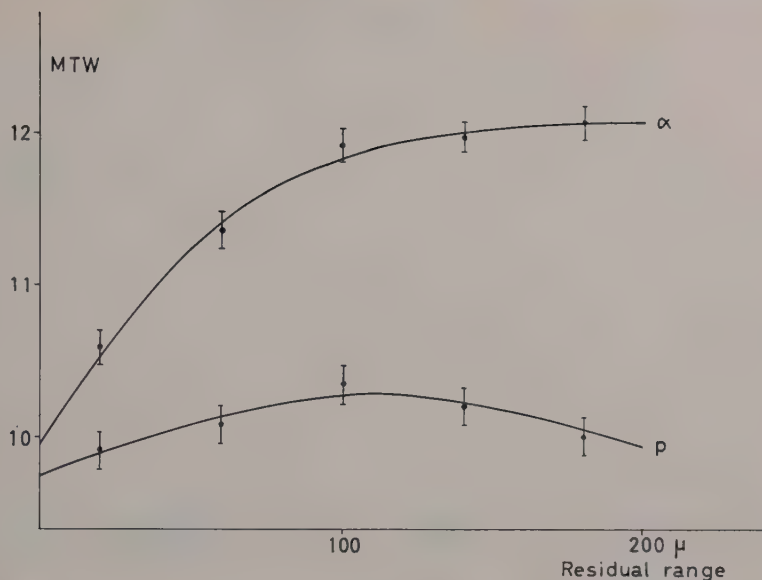


Fig. 8. MTW as a function of the residual range for protons and α -particles. Each curve is based on measurements of 18 particles. A maximum in MTW is found on the proton curve at $R = 110 \mu$.

Table I

MTW	σ (MTW)
10	0.090
11	0.078
12	0.066
13	0.060

one property is readily seen from the figure. There is a maximum in the MTW somewhere around $R = 90 \mu - 140 \mu$. If a function of the second degree is fitted to the experimental points, the maximum is found to occur at $R = 110 \mu$ corresponding to $\beta = 0.09$, which is in agreement with results found by others (4). The explanation of the occurrence of the maximum may possibly be the following simple one. At this speed of the particle it gives rise to a not negligible number of low energy δ -rays. If the energy of a δ -ray is, say 4 keV, it will give on the average one grain outside the core of the track. The number of δ -rays per 10μ with energy $> E_0$ (keV) is (12)

$$n_{\delta} = \frac{5.34 \cdot 10^{-4} \cdot Z^2}{\beta^2} \cdot \left[\frac{510}{E_0} - \frac{1}{2\beta^2} \right] \quad (3)$$

Z = charge of the particle, $\beta^2 < 1$.

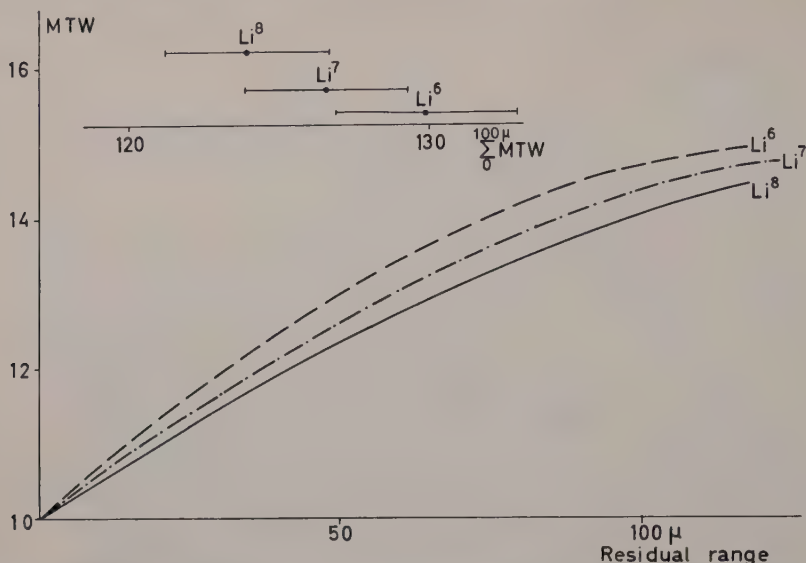


Fig. 9. MTW-residual range relations for the lithium isotopes Li^6 , Li^7 and Li^8 . The areas under the curves ($= \sum_{0}^{100 \mu} \text{MTW}$) will be slightly different for these isotopes. The expected values of $\sum_{0}^{100 \mu} \text{MTW}$ for the isotopes are indicated in the diagram where also the standard deviations of these sums have been indicated.

If we choose $E_0 = 4 \text{ keV}$, the maximum in n_δ is found at $\beta = 0.09$ and for protons the number of δ -rays at this velocity is about 4.4 per 10μ , which is enough to explain the increase in MTW at $\beta = 0.09$. At greater values of β the number of δ -rays decreases slowly. Of more importance then for the decrease in the total number of grains in the track is the lower primary energy loss.

The presence of this maximum can in theory be used for a rough mass determination since the range at which it occurs is proportional to the mass of the particle. Unfortunately the effect is small and the statistical spread of the measurements is of the same magnitude so that a mass estimate from this maximum is but of dubious value. Only when the residual range is long enough, which may be the case for deuterons and tritons for example, would it perhaps be possible to tell them from lighter particles.

Influence of different isotopes

The fact that the grain density range relation for particles with the same charge is also a function of the mass influences the quantity $\sum \text{MTW}$ used for the identification. The result is an additional spread in the total track area above the statistical variation. For $Z = 1$ it is negligible, the difference in $\sum_{0}^{100 \mu} \text{MTW}$ between protons and deuterons being less than 1 %. For $Z = 2$ the difference in $\sum_{0}^{100 \mu} \text{MTW}$ between He^3 and He^4 is approximately the same as the statistical spread ($\approx 3 \%$). However, if

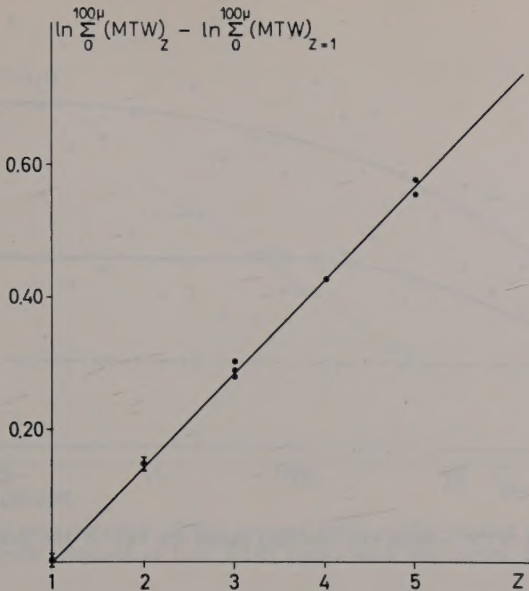


Fig. 10. $\ln \left(\sum_0^{100\mu} \text{MTW} \right)_Z - \ln \left(\sum_0^{100\mu} \text{MTW} \right)_{Z-1}$, as a function of the charge for tracks having $1 \leq Z \leq 5$.

the isotopes of lithium are considered, the effect is somewhat greater ($\simeq 4.5\%$ for Li^6 and Li^8) than the uncertainty in the total track area ($\simeq 2.5\%$). In Fig. 9 the MTW is shown as a function of the residual range for Li^6 , Li^7 and Li^8 , and in the same figure the $\sum_0^{100\mu} \text{MTW}$ and their uncertainties for the different lithium ions have also been indicated.

The reason for the lower spread in MTW for these tracks is the large number of grains in the tracks, which results in a standard deviation of only about 6.5% in the MTW. Whereas in many cases it seems just possible to distinguish between two isotopes such as Li^6 and Li^8 , B^8 and B^{11} etc., the method with the present emulsion (G 5) does not seem to be able to provide an answer to the interesting question as to which isotopes of He are present.

Determination of charges higher than three

In the emulsions originally used in this investigation on short tracks no particles with $Z > 4$ stopping in the emulsion could be found. Consequently, in order to check whether it is possible to identify charges higher than 3, a few plates of a stack of emulsions flown over Texas were scanned for tracks made by particles with charge ≥ 3 . A few singly charged and α -particles were selected for the sake of comparison. The number of background grains in these emulsions was very large. For very dense tracks this is, because of the large number of grains in the tracks, not so unfavourable for the measurements on particles with $Z \geq 3$ as it may seem. Therefore, for $Z \geq 3$ the statistical variation of $\sum \text{MTW}$ was much less than the differences to be measured, and charge determination was still possible.

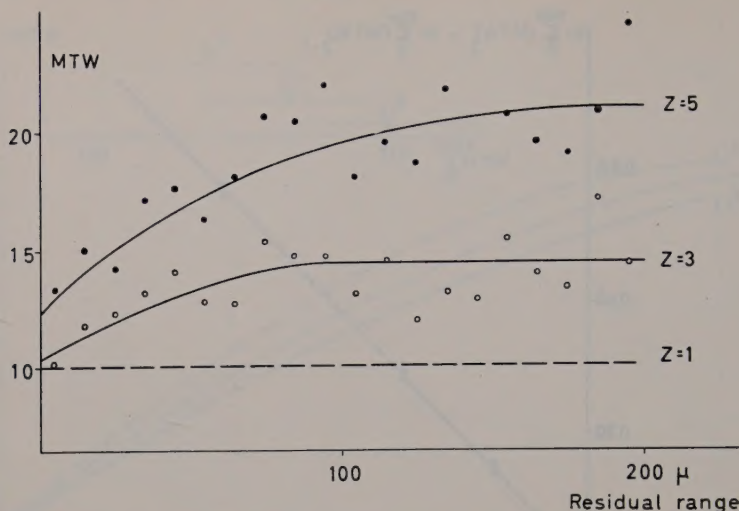


Fig. 11. A plot of MTW versus the residual range for two of the particles in Fig. 10.

Fig. 10 shows $\ln \left(\sum_0^{100 \mu} \text{MTW} \right)_Z - \ln \left(\sum_0^{100 \mu} \text{MTW} \right)_{Z=1}$ as a function of the assumed charge.

As in Fig. 6 the experimental points are to a fair approximation situated on a straight line. This empirical relation has also been found by Waldeskog (6), who used the same method of identification in his study of heavy primaries in the cosmic radiation. In diagram 10 one of the points refers to an identified B^8 and therefore the straight line relation seems to be well established up to $Z = 5$ and is probably valid to about $Z = 10$.

Criterion of a particle coming to rest before decaying or interacting

In Fig. 11 we can see the MTW as a function of the residual range for two of the particles in diagram 10. Both tracks seem to have the following properties in common:

1. MTW decreases markedly towards the ends of the tracks. This is due to the gradual loss of charge and the decreasing maximum range of the δ -rays.

2. The average spread of the individual measurements is less near the end since at low speed of the particle there are no one grain δ -rays outside the core of the track.

If the charges are high enough, say $Z \geq 3$, these two effects can be used as a criterion of a particle coming to rest before decaying or interacting. The eventual residual range can be estimated from the tapering process in the following way. A curve is fitted to the experimental points. Through the known MTW at $R = 0$ (the mean value of comparison particles of the same charge) a horizontal line is drawn. The curve intersects this line. If the range coordinate of this intersection is approximately zero, it is very likely that the particle has stopped or has a very short residual range. In the present material the difference in R was always less than 10μ and the reproducibility in the drawing good, about 5μ . All tracks showing the decrease in MTW towards the end of the range have also the characteristic decrease in the spread of the MTW. This seems a good indication of a particle slowing down. Of the two

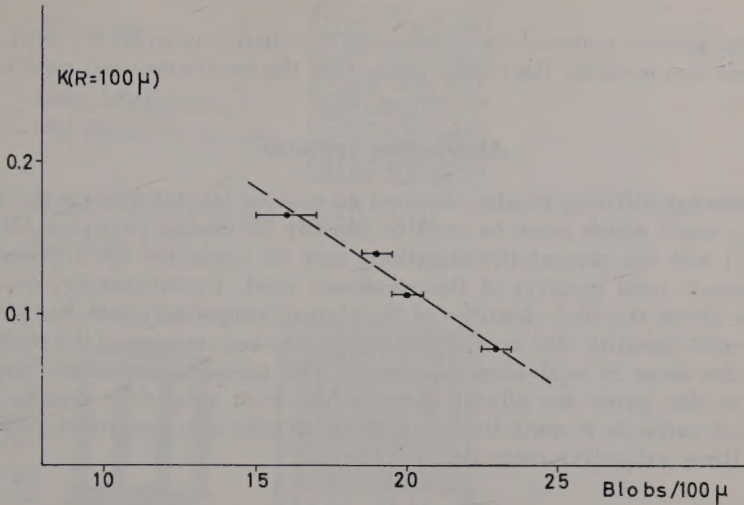


Fig. 12. $K(R = 100 \mu)$ as a function of the plateau blob density in G 5 emulsions. The broken line gives a crude estimate of how $K(R)$ varies with the degree of development.

effects the change in MTW is the most important because it provides a check that a particle has stopped or puts an extreme upper limit to its residual range (13). But it must be remembered that if the particle has a residual range greater than a certain value it is very difficult to provide an answer to the question of the true residual range.

Degree of development and resolution

The empirical relation given in formula (1) can be written

$$\left(\sum_0^R \text{MTW} \right)_Z = \left(\sum_0^R \text{MTW} \right)_{Z=1} \cdot e^{K(R) \cdot (Z-1)} \quad (4)$$

It means that the ΣMTW values for different charges form a geometrical series with $e^{K(R)}$ as the quotient i.e. as a measure of the relative distance between the groups containing particles with different charges. Evidently $K(R)$ is a function of the degree of development of the emulsions. In Fig. 12 $K(R)$ has been plotted versus the blob density of the plateau value. The points at the blob densities 19 and 20 refer to the measurements described in this paper, the point at 23 blobs/100 μ is taken from the measurements in connection with the particle described in reference (13), and the point at 16 is from the measurements by Waldeskog (6) (recalculated for a residual range of 100 μ).

From the diagram it can be seen that when the blob density decreases, $K(R)$ rapidly increases. Since the error in Z is proportional to $\frac{\sigma(\text{MTW})}{K(R)}$, the error in Z will be less when $K(R)$ is increased, that is when the emulsions are less developed. On the other hand there will be a slight increase in the relative standard deviation, $\sigma(\text{MTW})$, but it should remain small as long as the number of grains in the tracks is relatively

large. In the present material no estimate of the change in $\sigma(\text{MTW})$ with degree of development was possible, the reason being that the emulsions used were of varying quality.

Concluding remarks

The somewhat differing results obtained at various laboratories on the minimum length of a track which must be used to identify its charge (compare for instance (5) with (7) and the present investigation) may be explained by different degrees of development (and quality) of the emulsions used. Unfortunately, as a rule no statements about the blob densities of the plateau values are made, but since essentially the same quantity (the area of the tracks) has been measured the results should be about the same in equivalent emulsions. The measurements with the method described in this paper are almost entirely free from subjective factors, and this should be of value as it must be very difficult to take into account the errors introduced by these subjective estimates.

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